Structured and soft Boltzmann machine and mean-field approximation for structured sparse representations

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Abstract—We address the problem of structured sparse representation within a Bayesian framework. In particular, we consider a mean-field approximation for the estimation of the dependencies between atoms using a Boltzmann machine. This algorithm is shown to outperform the reference algorithm [1] with regard to their success criterion.

Index Terms—Structured sparse representations, Boltzmann machine, mean-field approximation.

I. INTRODUCTION

Recent contributions have emphasized the interest of considering structures between atoms selected in sparse representations (SR), for a wide range of dictionaries and classes of signals. This problem can be set into a Bayesian framework, e.g. Cevher et al. [2] and Faktor et al. [1]. Both use Boltzmann machines to model the dependencies between atoms, but differ in the prior model on the SR coefficients. In this paper, we consider a similar model as in [1].

Our observation model is \( y = \sum_{i=1}^{n} s_i x_i d_i + n \), where \( s_i \in \{0,1\}^M \) is the SR support, \( n \sim N(0, \sigma^2_I I_N) \) and \( I_N \) the identity matrix. We suppose that \( \forall i, p(x_i) = N(0, \sigma^2_s) \) and \( s \) is distributed according to a Boltzmann machine of parameters \( b \) and \( W \):

\[
p(s) \propto \exp(2b^T s + 2s^T W s - 2\sum_{t=1}^{M} W s_t)
\]

where \( I_M = \{1, \ldots, 1\} \) of length \( M \).

II. STRUCTURED SOFT BAYESIAN PURSUIT ALGORITHM

Based on this model, we consider here the following marginalized maximum a posteriori (MAP) estimation problem:

\[
s = \arg \max_{s \in \{0,1\}^M} p(s|y),
\]

where \( p(s|y) = \int p(x,s|y)dx \). To tackle problem (2), a greedy algorithm could be used [1] to approach the solution with a succession of local decisions. In this paper, we alternatively propose a mean-field algorithm to reconstruct the SR support via the probability

\[
1 - \frac{1}{K} \sum_{k=1}^{K} \max(\|s^{(k)}\|_0, \|s^{(k)}\|_0).
\]

The data is generated with \( N = 64, M = 256 \), and a DCT dictionary. The parameters are drawn independently: the elements of \( b \) from \( N(-2.5, 1) \) and the elements of \( W \) from \( U[-0.1, 0.1] \). The standard deviations \( \sigma_s \) are i.i.d. realizations of \( U[15, 60] \). For each point of simulation, we run 500 trials. We adjust the final threshold at 0.25. The figure above compares 2 algorithms: “MAP-greedy”, proposed in [1] and “SSoBaP” (for Structured Soft Bayesian Pursuit algorithm), proposed here. For the performance criterion considered, we can see that “SSoBaP” outperforms “MAP-greedy” over a wide range of noise variances.

IV. CONCLUSION

In this paper, we have shown that a MF approximation together with a VB-EM algorithm is a promising and competitive approach for the estimation of structures between atoms. To the extent of the considered criterion, the resulting algorithm is shown to outperform the baseline algorithm [1]. Complementary results, involving other performance criteria and other state-of-the-art algorithms, will be added in the final paper to confirm the relevance of this approach.

REFERENCES

